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AN OVERVIEW OF FOURIER TRANSFORM ON IMAGE PROCESSING

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Abstract:

The recognition of an images are important in the digital image processing. In this paper we introduce the definition of Fourier Transform and it's properties through which the solution of the problem will be easier than expected and also describe that what is the roll of Fourier transform in image recognition.

Keywords: Fourier transform (FT), Discrete Fourier transform (DFT), Image processing, Image recognition.

Introduction:

An image is a combination of physical likeness of objects, colours, or things photographed or painted. It is very easy to identify for a person like you and we to recognize similar or differential objects in the given image i.e. to identify flower in the image, animal in the image. Identification and recognition of the objects in the image is very easy for human eye, but it may not be simple for a machine without the help of human interruption to identify and recognize objects in the image because sometimes we don't have labels and headings in the Image, but we want to detect and recognize objects, colours or like things in the image. Image recognition is a type of technique through which we can identify and recognize objects, places, actions in any given image, logos and several other variable or content in digital image. So we can use a tool in the machine to recognize the objects, colours or like things with its position and characteristics in the image i.e. Fourier transform.

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Fourier transform is came from the name of French military scientist , French Baron Jean – Baptiste-Joseph Fourier , after year of research he uncovered this powerful tool in the early 1800s [1]. It is primary invented and used for physics equation and later on same tool is now widely used in all over the world for Mathematical science, signal processing , image analysis, image filtering, image reconstruction and image compression.[2]

Fourier transform is not only used for understanding the nature and characteristics of the image but also is to be used for processing the same, it executes a procedure on the image in which image it decompose its components into sine and cosine components.

The processed output of the transformation represents (H. B. Kekre, T. Saroode & P. Natu, 2014) the image in the frequency domain and the image on which the procedure of Fourier transform is executed is on spatial domain equivalent.

In current era, the use of digital images has increased day by day(A. M Eslicioglu & P. S. Fisher, 1995; T. Kin & X. P. Zhang 2008). Recently image recognition is attractive much attention in the society of network multimedia information access. In the digital area the technology of image recognition is the process of identifying and detecting a feature of an image or video. Reorganization of an image is a representative and convenient information in the field of image processing. Image recognition is the ability of a system or software to identify places, people, items, buildings, logos, different object in an image by the help of machine vision technology with trained algorithm [9]. Many companies like Facebook, Apple, Google, Microsoft have invested heavily in research to develop image recognition and its uses. The main benefit of Fourier Transform is that very little information is lost form the signal during the Transformation.

Definition of Fourier Transform:

The Fourier Transform is a fundamental importance in (A. Mcandrew, 2004) image processing tool which used to decompose an image into its sine and cosine components [6].

But before we venture into understanding Fourier transform on an image, let us first look into the Fourier transform of a continuous valued one-dimensional signal[4]. There are several common conventions for defining the Fourier transform of an integrable function f: R → C and the formulae used to defined Fourier Transform vary according to different authors (Arfken, 1985, Krantz,1999 and Trott 2004). But they are essentially the same but using different scales. One of them is:

One dimension Fourier Transform

The one dimension continuous Fourier Transform (CFT) is a continuous function f(x) (A. Graphs, 1996, R. J. E. Merry, 2005; J. C. Goswami & A. K.Chan, 2011)

$$\boldsymbol{F}(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-i2\pi\omega x}\, dx, \qquad \forall\ \omega \in R. \qquad \text{(Eq.1)}$$

where $i=\sqrt{-1}$ and $e^{i\theta} = \cos\theta + i\sin\theta$

The transform of function $f(x)$ at frequency ω is given by the complex number

$\boldsymbol{F}(\omega)$.

Fourier inversion integrals

$$\boldsymbol{f}(x) = \int_{-\infty}^{\infty} F(\omega)\, e^{i2\pi\omega x}\, d\omega, \qquad \forall\ x \in R. \text{ (Eq.2)}$$

which, for the real-valued $\boldsymbol{f}(x)$, reduces to:

$$\boldsymbol{f}(x) = 2\int_0^{\infty} Re\left(F(\omega).\, e^{i2\pi\omega x}\right) d\omega$$

$$=2\int_0^{\infty}\left(Re\big(F(\omega)\big).\ \cos(2\pi\omega x) - Im\big(F(\omega)\big).\ \sin(2\pi\omega x)\right) d\omega .$$

The complex number, $\boldsymbol{F}(\omega)$, conveys both amplitude and phase of frequency ω. Eq2 is kuown as the Fourier inversion therorem, and was first introduced in Fourier Analytical theory of Heat. (1. Fourier, J.B. Joseph (1878) [1822], The Analytical Theory of heat translated by Alexander Freeman, The University Press(translated from French).

Two dimensional Fourier Transform-

Extending the concept of one dimensional Fourier Transform, The analysis and synthesis formulas for the 2D continuous Fourier Transform are as follows:

Analysis

$$f(x,y) = \int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty} F(p,q)e^{i2px}\, e^{i2qy} dx\, dy \qquad \text{(Eq.3)}$$

the spatial function $f(x,y)$ is decomosed into a weighted sum of 2D orthogonal basis functions in a similar manner to decomposing a vector onto a basis using scalar products.

Synthesis

The corresponding inverse two dimensionl Fourier Transform is

$$F(p,q) = \int_{q-\infty}^{\infty}\int_{p-\infty}^{\infty} F(x,y)e^{-i2px}\, e^{-i2qy} dx\, dy \qquad \text{(Eq4)}$$

Discrete Fourier Transform (DFT)

When the function is represented in discrete from using a sequence of discrete sample such as f(x) = { f(0), f(1),............., f(N-1), the corresponding Fourier Transform of the discrete signal is the Discrete Fourier Transform (DFT) [5].

One- dimension discrete Fourier Transform

· Forward

The one-dimension discrete Fourier Transform from of a function f(x) of size N with integer index x running from 0 to N-1, is represented by

$$F(u) = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} f(x).\left[\cos\left(2\pi\frac{ux}{N}\right) - i.\sin\left(2\pi\frac{ux}{N}\right)\right]$$

$$F(u) = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} f(x).e^{\frac{-i2\pi ux}{N}} \qquad 0 \le u \le N$$

· inverse

$$f(x) = \frac{1}{\sqrt{N}} \sum_{u=0}^{N-1} f(u).\left[\cos\left(2\pi\frac{ux}{N}\right) + i.\sin\left(2\pi\frac{ux}{N}\right)\right]$$

$$f(x) = \frac{1}{\sqrt{N}} \sum_{u=0}^{N-1} f(u).e^{\frac{i2\pi ux}{N}} \qquad 0 \le u \le N$$

M is the lenghth (number of discrete samples)

Two-dimension Discrete Fourier Transform

· Analysis equation

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

where u=0, 1, 2.........M-1 and v= 0, 1, 2,.........N-1

· Synthesis equation

$$F(x,y) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(u,v)e^{i2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

where x= 0, 1, 2........M-1 and y= 0, 1, 2.........N-1

Discrete Fourier Transform is also atrective for image recognition. The DFT is the transform which takes the discrete signal in the time domain and transform that signal in its time domain transform that signal in its discrete frequency domain [7].

Properties in fourier Transform are:- Linearity, differentation and integration, time and frequency scaling, duality, time shifting parseval's relation; polar representation, magnitude and phase, Bode plots; Convolution and modulation properties and the basis they provide for filtering, modulation, and sampling; Use of transform methods to analyze LTI systems characterized by differential and differece equations; Cascade and parallel from realization: first- and second- order systems.

Some properties are:

• Linearity: a$x_1(t) + bx_2(t) \xrightarrow{FT}$ a$X_1(\omega)$ + b$X_2(\omega)$

where a and b are constant

•Differentiation in Time:

X(t) ↔ X(jω)

$\frac{d^k x(t)}{dt^k} \leftrightarrow (j\omega)^k X(j\omega)$

• Integration in Time:

X(t) ↔ X(jω)

$\int_{-\infty}^{t} X(t)dt \leftrightarrow \frac{X(j\omega)}{j\omega} + \pi \times (0)\delta(\omega)$

• Translation:

F$[f(x - x_{0,}\ y - y_0)]$ = F(u, v) $e^{-i2\pi\left(\frac{ux_0}{M}+\frac{vy_0}{N}\right)}$

The fourier Spectrom remains unchanged under translation:

$|F(u,v)|=\left|F(u,v)e^{-i2\pi\left(\frac{ux_0}{M}+\frac{vy_0}{N}\right)}\right|$

• Rotation:

Rotation of f(x,y) by 0 → rotation of F(u,v) by θ

• Scaling:

F$[f(ax,by)]$ = $\frac{1}{|ab|}$F$\left(\frac{u}{a},\frac{v}{b}\right)$

• Duality:

x(t)↔X(ω)

X(t) ↔ $2\pi x(-\omega)$

Application of Fourier Transform

Fourier Transforms methods are use in the fields of technology and science, such as Application of IBVP, Circuit Analysis, Cell phone, Signal analysis, Image processing, Signal Processing and LIT system, image compression and other areas. Using Fourier transform, it has been possible to analyze an image as a set of spatial sinusoids in various directions, each sinusoid having a precise frequency.[3] Fourier Transform has many application in the field of physical science that uses sinosodal signals, such as engineering, phyisics, applied mathematics, and chemistry, will make use Fourier series and Fourier Transform.

Fourier Transform help to analyze spectrom of the signal, helps in find the response of the TIL system (continuous time fourier transform is for analog signal and Discrete time fourier transform is for discrete signals) [8].

Uses of image recognition in our daily rutine

The use of image reognition are found in different field such as healthcare, marketing, transformation and e-commerce. Image recognition can be used to identify objects in image to categorize them for future use.

Some common uses of image recognition that we see in our daily lives:-

1. Modern smart phones uses facial recognition as feature for unlock phones and this feature is highly useful in case phone is stolen or missing. This feature not only unlock phones but very helpful in protecting data and secure data from being accessed.

2. Investigation agencies uses image recognition to collect data regarding history of suspected , they takes phtograph and search with their database. Investigation agencies also scanned and add photgraphs to carry out another search

3. Image recognition technique is also used in international aerodrems and ports for travellers having biometric passports for facial recognition, it not only saves times but also improve safety and security .

4. Image recognition method is also used in reverse image search on web. In combination with artificial intelligence software we can search and identify image features and compare with the similar images available on web , able to find out the origin of the image , ownership of the image and all the relevant information of the image available from different sources.

5. Image recognition is also very helpful in e commerce industry. Online customers are more willing to search products by their image instead of using text search for their purchases.

6. Attendence is very important and compulsory process in each and every oganisation. Automatic attendance system uses image recognition technique and generate data accurate and real time.

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