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A NEW APPROACH FOR FINDING AN INITIAL SOLUTION NEAR TO OPTIMUM FOR THE SOLID TRANSPORTATION PROBLEMS

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Abstract

The solid transportation problem (STP) is a particular type of linear programming problem. This paper presented an approach for solving STP in a highly efficient and a few iterations until finding an optimal solution. The proposed method provides an initial solution near to optimum solution to the solid transportation problem close to the optimal solution. Using Lingo software, a numerical example was used to compare the solution from the proposed method to the optimal solution.

Introduction

Solid transportation problem (STP) is a particular type of linear programming problem (LPP) [1]. In a transportation problem, we have some sources and destinations. The aim is to distribute a product from sources to destinations for fulfilling supply and demand. In STP, we have at least one primary dimension more. This dimension is the existence of a difference in the types of transportation by which products are transported from sources to destinations. In this problem, we have three types of constraints that are considered [2]:

A. Source constraints in terms of product availability
B. Destination constraints in terms of product demand
C. Transportation fleet capacity constraints

Haley [3] explained that the solid transport problem was formulated first by Shell in a research paper entitled "Distribution of a product by several properties," but he is considered the first to formulate a method to solve the STP as it made an extension of the Modified Distribution

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Method (MODI) used to find the optimal solution to solve the classical transportation problem and found through which an optimal solution to the problem of solid transportation. The paper explained the difference between both problems and how to find the initial basic solution and the optimal solution for STP. The research explained how to calculate shadow costs, which is the most critical step in the (MODI method) to reach an optimal solution. The method was characterized by its speed of finding the solution and its ability to solve large-scale problems. It was the first step in finding more than one other approach to solving STP.

Pandian et al. [4] formulated a new approach to solve the 3D solid transportation problem. This method is based on a modification of the zero-point method used to solve the classical transportation problem. A solution algorithm for the proposed approach was made and tested with a numerical example solution that solved the STP equivalent to the optimal solution. The advantage of this paper is that the author solved the problem using the MODI method, which means faster problem-solving, especially with large-scale problems. Solving the problem in this way requires basic variables whose number is m+n+l-2, where m is the number of sources, n is the number of destinations, and l is the number of types of trucks used for transport. Munot et al. [5] formulated an approach to solving the solid transportation problem called GM Method. In this approach, Ghadle-Munot Algorithm is used. The problem with three restrictions is solved by considering three different forms: Supply-Demand-Conveyance, Supply-Conveyance-Demand, and Conveyance-Demand-Supply by a specific solution algorithm. And the objective function or the least cost among the previous three forms is considered the best answer, which gives a more efficient solution. Finally, a code was created using the MATLAB program to solve the problem.

This paper presents a new approach to finding an Initial solution close to optimal for STP called the "Avoiding the Bigger Cost" ABC method. This method eliminates cells that have a high transportation cost unit, which leads to a reduction in the total transportation cost. A solution algorithm has been introduced to demonstrate how to solve this approach. A numerical example was also solved to clarify the solution algorithm and compare them with the optimal solution.

The General Structure of the Solid Transportation Problem

A generalization of the classical transportation problem (CTP) is the solid transportation problem (STP). The objective function and constraint sets consider three-dimensional properties in place of destination and source. A similar product is shipped from a source to a destination via various conveyances or modes of transportation, such as ships, freight trains, cargo aircraft, trucks, etc. These means of transportation enable us to represent the problem in three dimensions. The STP can be turned into a CTP if we take into account one mode of transportation. [6].

The mathematical form of solid transportation problem is given by [7]:

$$\text{Min } Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p} c_{ijk}x_{ijk} \quad (1)$$

Subject to:

$$\sum_{j=1}^{n}\sum_{k=1}^{p} x_{ijk} = a_i \qquad i = 1, 2, ..., m \quad (2)$$

$\sum_{i=1}^{m} \sum_{k=1}^{p} x_{ijk} = b_j$ j = 1, 2, ..., n (3)

$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = e_k$ k = 1, 2, ..., p (4)

$x_{ijk} \geq 0 \; for \; all \; j, j, k$ (5)

Where:

z = objective function

m = number of sources of the STP

n = number of destinations of the STP

p = number of different modes of the STP

x_{ijk} = the amount that transported from source i to destination j by conveyance k

c_{ijk} = unit transportation cost in STP

a_i = amount of products available in source i

b_j = demand at destination j

e_k = the amount of product that can carried by conveyance k

ABC Method Solution Algorithm

Step 1:

1. Check if the transportation problem is balanced or not.
   i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{p} e_k$
1. If it unbalanced, add a dummy source or destination or convenyance with an availability, requirement or capacity. Now, go to step 2

Step 2:

1. The transportation problem matrix includes (m*n*p) cells, where "m" represents the total supply and "n" represents the total demand and "p" the total transportation fleet capacity.
2. Based on that find the cube root of the (m*n*p). (i.e$\sqrt[3]{m * n * p}$).
3. Sort the unit costs in ascending order.
4. Prepare a sample of "the lowest" unit costs, which equals $\sqrt[3]{m * n * p}$.
5. If there are more unit costs have same lowest values, the sample should include all of them.
6. Consider the minimum of the chosen unit costs of this sample to start with. Go to step 3

Step 3:

1. Check whether the chosen cell's source availability, destination requirement, or truck capacity has a lower value.
2. Calculate the sum of the other unit costs for the row or column with the lower value. This should not include the chosen unit cost.
3. Repeat "Step 3" for the next unit cost of the chosen sample of unit costs. If all unit costs in the chosen sample are considered, go to "Step 4".

Step 4:

1. Sort the calculated cells in "Step 3 - point 2" descending.
2. Choose the cell of the sample which is corresponding to the biggest calculated value.

3. Fill this cell with the minimum of its source availability, its destination requirement and its capacity.
4. Consider the cell with the next biggest calculated value and go to "Step 4 – point 3."
5. If all the calculated values are considered, go to "Step 5".

Step 5:

a. Now either the transportation table cells are entirely filled, or there may be values that still need to be filled if they are all filled stop.
b. If the remaining cells are four or more "not in one row or one column," go to "Step 2 – point 2" to prepare a new sample based on the remaining number of cells.

Numerical Example

An example has also been resolved in detail to show how to solve the solid transportation problem with the new approach.

Table (1) illustrates a transportation problem example including all availabilities, requirement conveyances, and the transportation cost.

										capacity
	E1			E1			E1			30
		E2			E2			E2		30
			E3			E3			E3	40
	D1			D2			D3			Avail.
S1	1	4	5	7	8	3	4	7	6	20
S2	3	5	6	6	7	1	9	4	8	40
S3	6	8	5	4	5	9	6	2	3	40
Req.	32			24			44			

- First, take a sample ($\sqrt[3]{m * n * p}$) as shown in step 2 and step 3 to select the cell to start with.

Table (2) testing selected values

Cell	value	source	Destination	Mode	Min.	Priority
C111	1	44	42	45	44	3rd
C223	1	48	49	45	45	2nd
C332	2	46	47	48	46	1st

Then start filling in the cells that have priority for filing first and solve the problem as in steps 4 and step 5.

Table (3) making the first allocation

										capacity	
	E1			E1			E1			30	
		~~E2~~			~~E2~~			~~E2~~		~~30~~	~~0~~
			E3			E3			E3	40	

	D1			D2			D3			Avail.	
S1	1	4	5	7	8	3	4	7	6	20	
S2	3	5	6	6	7	1	9	4	8	40	
S3	6	8	5	4	5	9	6	2 **(30)**	3	~~40~~	10
Req.	32			24			~~44~~				
							14				

Table (4) making the second allocation

										capacity	
	E1			E1			E1			~~30~~	~~6~~
		~~E2~~			~~E2~~			~~E2~~		~~30~~	~~0~~
			E3			E3			E3	40	
	D1			D2			D3			Avail.	
S1	1	4	5	7	8	3	4	7	6	20	
S2	3	5	6	6	7	1 **(24)**	9	4	8	~~40~~	16
S3	6	8	5	4	5	9	6	2 **(30)**	3	~~40~~	10
Req.	32			~~24~~			~~44~~				
				0			14				

Table (5) making the third allocation

										capacity	
	E1			E1			E1			~~30~~	10
		~~E2~~			~~E2~~			~~E2~~		~~30~~	~~0~~
			E3			E3			E3	~~40~~	16
	D1			D2			D3			Avail.	
S1	1 **(20)**	4	5	7	8	3	4	7	6	~~20~~	0
S2	3	5	6	6	7	1 **(24)**	9	4	8	~~40~~	16
S3	6	8	5	4	5	9	6	2 **(30)**	3	~~40~~	10
Req.	~~32~~			~~24~~			~~44~~				
	12			0			14				

As a result of filling all the cells that were selected in the sample, we make a small sample of the remaining cells to determine which cells to start filling in again, and we repeat steps 1 and 2 again.

Table (6) testing remaining values

Cell	value	source	Destination	Mode	Min.	Priority
C211	3	29	17	27	17	1st
C333	3	17	23	19	17	1st

Then start filling again in the cells that have priority for filing first and solve the problem as in steps 4 and step 5.

Table (7) making the fourth allocation

										capacity		
	E1			E1			E1			~~30~~	~~10~~	~~0~~
		~~E2~~			~~E2~~			~~E2~~		~~30~~	~~0~~	
			E3			E3			E3	~~40~~	~~16~~	6
	D1			D2			D3			Avail.		
S1	1 **(20)**	4	5	7	8	3	4	7	6	~~20~~	0	
S2	3 **(10)**	5	6	6	7	1 **(24)**	9	4	8	~~40~~	~~16~~	6
S3	6	8	5	4	5	9	6	2 **(30)**	3 **(10)**	~~40~~	10	0
Req.	~~32~~			~~24~~			~~44~~					
	~~12~~			0			~~14~~					
	2						4					

Table (7) making the final allocation

										capacity			
	E1			E1			E1			~~30~~	~~10~~	~~0~~	
		~~E2~~			~~E2~~			~~E2~~		~~30~~	~~0~~		
			E3			E3			E3	~~40~~	~~16~~	~~6~~	0
	D1			D2			D3			Avail.			
S1	1 **(20)**	4	5	7	8	3	4	7	6	~~20~~	0		
S2	3 **(10)**	5	6 **(2)**	6	7	1 **(24)**	9	4	8 **(4)**	~~40~~	~~16~~	6	
S3	6	8	5	4	5	9	6	2 **(30)**	3 **(10)**	~~40~~	10	0	
Req.	~~32~~			~~24~~			~~44~~						
	~~12~~			0			~~14~~						
	2						4						
	0						0						

Therefore, the solution to the given solid transportation problem is x111=20, x211=10, x213=2, x223=24, x233=4, x332=30, x333=10 and the objective value is 208

LINGO is software made specifically for quickly and effectively solving linear and nonlinear models and optimization models [8]. The software can address many issues, but it is somewhat challenging to use and expensive compared to others. The programme can solve transportation problems when the costs of transportation are expressed in real values rather than just integers, which can be challenging in other applications [9]. We used it to solve STP as a linear programming problem and get the optimal solution directly.

Tables (8 and 9) illustrate the Lingo code & Lingo output for an illustrative example which is subsequently solved.

Table 8. A typical Lingo code for an STP example.

```
Model:
Min = 1*X111 + 4*X112 + 5*X113 + 7*X121 + 8*X122 + 3*X123 + 4*X131 + 7*X132 +
6*X133 + 3*X211 + 5*X212 + 6*X213 + 6*X221 + 7*X222 + 1*X223 + 9*X231 + 4*X232
+ 8*X233 + 6*X311 + 8*X312 + 5*X313 + 4*X321 + 5*X322 + 9*X323 + 6*X331 +
2*X332 + 3*X333;
X111 + X112 + X113 + X121 + X122 + X123 + X131 + X132 + X133 = 20;
X211 + X212 + X213 + X221 + X222 + X223 + X231 + X232 + X233 = 40;
X311 + X312 + X313 + X321 + X322 + X323 + X331 + X332 + X333 = 40;
X111 + X112 + X113 + X211 + X212 + X213 + X311 + X312 + X313 = 32;
X121 + X122 + X123 + X221 + X222 + X223 + X321 + X322 + X323 = 24;
X131 + X132 + X133 + X231 + X232 + X233 + X331 + X332 + X333 = 44;
X111 + X121 + X131 + X211 + X221 + X231 + X311 + X321 + X331 = 30;
X112 + X122 + X132 + X212 + X222 + X232 + X312 + X322 + X332 = 30;
X113 + X123 + X133 + X213 + X223 + X233 + X313 + X323 + X333 = 40;
```

Table 9. Lingo code output for STP numerical example.

```
Global optimal solution found.
Objective value:                 196.0000
Infeasibilities:               0.000000
Total solver iterations:                7
                    Variable        Value
                       X111     20.00000
                       X112     0.000000
                       X113     0.000000
                       X121     0.000000
                       X122     0.000000
                       X123     0.000000
                       X131     0.000000
                       X132     0.000000
                       X133     0.000000
```

X211	10.00000
X212	2.000000
X213	0.000000
X221	0.000000
X222	0.000000
X223	24.00000
X231	0.000000
X232	4.000000
X233	0.000000
X311	0.000000
X312	0.000000
X313	0.000000
X321	0.000000
X322	0.000000
X323	0.000000
X331	0.000000
X332	24.00000
X333	16.00000

Row	Slack or Surplus
1	196.0000
2	0.000000
3	0.000000
4	0.000000
5	0.000000
6	0.000000
7	0.000000
8	0.000000
9	0.000000
10	0.000000

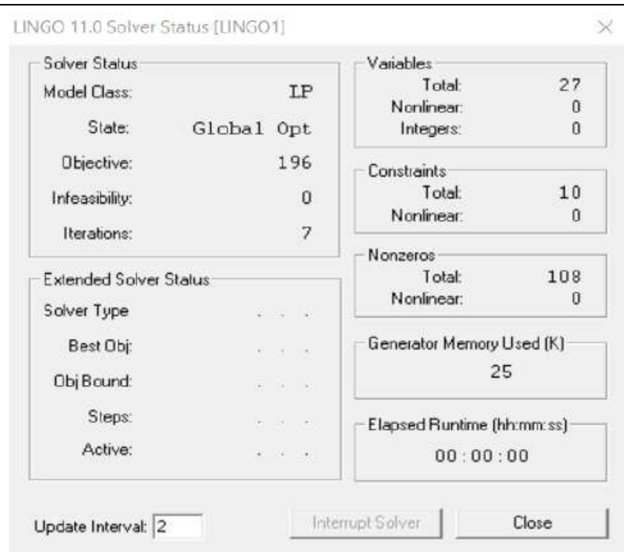

Fig. 1 shows the output of the code

Results and Discussion

After solving the previous numerical example using the ABC method and comparing it with the optimal solution produced by Lingo software, it was found that the proposed method gives an initial solution of 92% of the optimal solution. With large-scale solid transportation problems, this solution is more efficient and closer to optimal. The resulting solution was a basic feasible solution with m+n+l-2 nonzero values of the decision variables, which is required for STP to start with a basic feasible solution.

Conclusion

This paper presented a proposed approach to solve the solid transportation problem that is an improvement of the ABC method. The efficiency of the method has been demonstrated and compared with the optimal solution resulting from the software (Lingo Code). In light of the high cost of purchasing optimization and decision-making software, this method seems highly efficient in helping researchers and specialists find solutions acceptable to decision-makers with regard to STP.

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